Section 8.2
L'Hopital's Rule

Recall from Section 2.1

$$\lim_{x \to 0} (\sin x) / x = \frac{0}{0}$$

Recall (indeterminate forms)

$0/0$, ∞/∞, $\infty - \infty$, 1^{∞}, $0 * \infty$, 0^0, ∞^0

Cannot be found by just substituting a

If you weren't able to determine the limit

L'Hopital's Rule (First Form)

Suppose that $f(a) = g(a) = 0$, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$.

Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

OR

Suppose that $f(a) = g(a) = 0$, that f and g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to a} \ln f(x) = L \quad \Rightarrow \quad \lim_{x \to a} f(x) = \lim_{x \to a} e^{\ln (f(x)} = e^{L}$$

$$\ln \lim_{x \to \infty} f(x) = 1$$

$$\log_{e} \lim_{x \to \infty} f(x) = 1$$

$$\lim_{x \to \infty} f(x) = e^{1} = e$$

Examples

$$\lim_{x \to 0} \frac{x^2}{x^3 - x} = \frac{0}{0-0} = \frac{0}{0}$$

L'HOP

$$= \lim_{x \to 0} \frac{2x}{3x^2 - 1} = \frac{0}{-1} = 0$$

$$\lim_{x \to 0} \frac{\cos x - 1}{x} = \frac{1-1}{0} = \frac{0}{0}$$

L'HOP

$$= \lim_{x \to 0} \frac{-\sin x}{1} = \frac{0}{1} = 0$$

$$\lim_{x\to 2^{+}} \frac{x^2 - 4}{\frac{x^2}{2} - 2x + 2} =$$

$$\lim_{x = \frac{\pi}{2}} \frac{\sec x}{1 + \tan x} = \frac{\infty}{\infty}$$

L'HOP

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sec x \tan x}{\sec^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\tan x}{\sec x} = \lim_{x \to \frac{\pi}{2}} \frac{\frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1}}{\frac{1}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1}}$$

$$= \lim_{x \to \frac{\pi}{2}} \sin x = 1$$

$$\lim_{x\to\infty} xe^{-x} = \infty \cdot 0$$

$$= \lim_{x\to\infty} \frac{x}{e^x} = \frac{\infty}{\infty}$$

L

$$= \lim_{x\to\infty} \frac{1}{e^x} = \frac{1}{\infty} = 0$$

$$\lim_{x \to 5} \frac{\int_5^x t^2 + 1 \, dt}{2x - 10} = \frac{0}{0}$$

L

$$= \lim_{x \to 5} \frac{x^2 + 1}{2} = \frac{26}{2} = 13$$

$$\lim_{x \to 0} \frac{x^2 - 2x}{4x + x^3} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{x} =$$

$$\lim_{x \to 0} \frac{\sqrt{1 + x} - 1 - x/2}{x^2} = \frac{0}{0}$$

Using L'Hopital's Rule with One-Sided Limits

a.) $$\lim_{x \to 0^+} \frac{\sin x}{x^2} =$$

b.) $$\lim_{x \to 0^-} \frac{\sin x}{x^2} =$$

$$\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}} = \frac{\infty}{\infty}$$

$2x^{\frac{1}{2}}$

(L)

$$= \lim_{x \to \infty} \frac{\frac{1}{x}}{x^{-\frac{1}{2}}} = \lim_{x \to \infty} \frac{\frac{1}{x} \cdot \frac{\sqrt{x}}{1}}{\frac{1}{\cancel{\sqrt{x}}} \cdot \frac{\cancel{\sqrt{x}}}{1}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}}{x}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{x}} = 0$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = (1+0)^{\infty} = 1^{\infty}$$

$$\ln f(x) = \ln \left(1+\frac{1}{x}\right)^x$$

$$\ln f(x) = x \ln \left(1+\frac{1}{x}\right)$$

$$\lim_{x \to \infty} \ln f(x) = \lim_{x \to \infty} x \ln \left(1+\frac{1}{x}\right)$$

Find

a.) $\lim_{x \to \infty} (x \sin \frac{1}{x}) =$ $\infty \cdot 0$

$\lim_{x \to \infty} \dfrac{\sin \frac{1}{x}}{\frac{1}{x}} = \dfrac{0}{0}$

b.) $\lim_{x \to -\infty} (x \sin \frac{1}{x}) =$

$$\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x - 1} \right) =$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x =$$

$$\lim_{x \to 0^+} x^x =$$

$$\lim_{x \to \infty} x^{1/x} =$$